



## EE 232 Lightwave Devices Lecture 23: Optical Modulators

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Prof. Ming Wu



## Physical Mechanisms for Optical Modulators

- **Electro-optic modulators**
  - Nonlinear crystals
  - LiNbO<sub>3</sub>, GaAs, InP
- **Franz-Keldysh effect**
  - Sub-bandgap absorption induced by electric field
  - GaAs, InP
- **Quantum confined Stark Effect (QCSE)**
  - Absorption modulators in quantum wells
  - Mostly III-V, but also SiGe QWs
- **Free carriers effect**
  - Refractive index change due to electrons/holes
  - All semiconductors, including Si
- **Thermo-optic effect**
  - Refractive index change due to temperature
  - All semiconductors, including Si

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## Electro-Optic Effect (Pockels Effect)

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{41}F_x yz + 2r_{52}F_y zx + 2r_{63}F_z xy = 1$$

**Table 12.1 A Few Electrooptic Materials With Their Parameters [1, 4, 6, 9]**

Point-Group Symmetry	Material	Refractive Index		Wavelength $\lambda_0(\mu\text{m})$	Nonzero Electrooptic Coefficients ( $10^{-12}$ m/V)
		$n_o$	$n_e$		
$3m$	LiNbO <sub>3</sub>	2.297	2.208	0.633	$r_{13} = r_{23} = 8.6, r_{33} = 30.8$ $r_{42} = r_{51} = 28, r_{22} = 3.4$ $r_{12} = r_{61} = -r_{22}$ $r_{41} = -r_{52} = 0.2$ $r_{62} = r_{21} = -r_{11} = 0.93$
32	Quartz (SiO <sub>2</sub> )	1.544	1.553	0.589	$r_{41} = r_{52} = 8.77, r_{63} = 10.3$
$\bar{4}2m$	KH <sub>2</sub> PO <sub>4</sub> (KDP)	1.5115	1.4698	0.546	$r_{41} = r_{52} = 8, r_{63} = 11$
$\bar{4}2m$	NH <sub>4</sub> H <sub>2</sub> PO <sub>4</sub> (ADP)	1.5266	1.4808	0.546	$r_{41} = r_{52} = 23.76, r_{63} = 8.56$
$\bar{4}2m$	KD <sub>2</sub> PO <sub>4</sub> (KD*P)	1.5220	1.4773	0.633	$r_{41} = r_{52} = 23.41, r_{63} = 7.828$
$\bar{4}2m$	KD <sub>2</sub> PO <sub>4</sub> (KD*P)	1.5079	1.4683	0.546	$r_{41} = r_{52} = 8.8, r_{63} = 26.8$
$\bar{4}3m$	GaAs	3.60	$= n_o$	0.9	$r_{41} = r_{52} = r_{63} = 1.1$
		3.42	$= n_o$	1.0	$r_{41} = r_{52} = r_{63} = 1.5$
		3.34	$= n_o$	10.6	$r_{41} = r_{52} = r_{63} = 1.6$
$\bar{4}3m$	InP	3.29	$= n_o$	1.06	$r_{41} = r_{52} = r_{63} = 1.45$
		3.20	$= n_o$	1.35	$r_{41} = r_{52} = r_{63} = 1.3$
$\bar{4}3m$	ZnSe	2.60	$= n_o$	0.633	$r_{41} = r_{52} = r_{63} = 2.0$
$\bar{4}3m$	$\beta$ -ZnS	2.36	$= n_o$	0.6	$r_{41} = r_{52} = r_{63} = 2.1$

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## GaAs Electro-Optic Modulators

$$r = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{52} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$$

Apply electric field in z direction:

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{63}F_z xy = 1$$

$$n_{x'} = n_o + \frac{1}{2}n_o^3 r_{63} F_z$$

$$n_{y'} = n_o - \frac{1}{2}n_o^3 r_{63} F_z$$

For GaAs at  $1\mu\text{m}$  wavelength

$$n_o = 3.42, \quad r_{41} = r_{52} = r_{63} = 1.5 \times 10^{-12} \text{ m/V}$$

For applied field of  $10^7$  V/m

$$\Delta n = \frac{1}{2}n_o^3 r_{63} F_z \approx 3 \times 10^{-4}$$

Note: Si is central symmetric and has no electro-optic effect

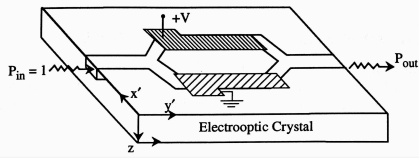
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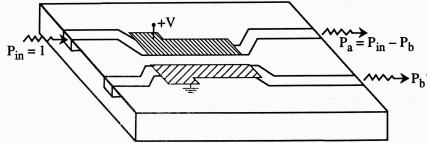


## Converting Index Change to Amplitude Change

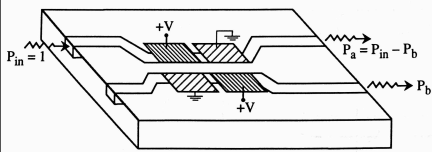
(a) A Mach-Zehnder interferometric waveguide modulator



(b) A directional coupler modulator



(c) A Δβ-phase-reversal directional coupler



$$I_o = |E|^2 = \left| \frac{1}{2} E \cdot e^{j \frac{2\pi}{\lambda} (n_o + \Delta n) L} + \frac{1}{2} E \cdot e^{j \frac{2\pi}{\lambda} (n_o - \Delta n) L} \right|^2$$

$$= |E|^2 \frac{1}{2} \left( 1 + \cos \left( \frac{4\pi \Delta n L}{\lambda} \right) \right)$$

$$= |E|^2 \frac{1}{2} \left( 1 + \cos \left( \frac{4\pi n_o^3 r_{63} F_z L}{\lambda} \right) \right)$$

$$F_z = \frac{V}{d}$$

$V_\pi$  : voltage at  $\pi$  phase shift

$$V_\pi L = \frac{d\lambda}{4n_o^3 r_{63}}$$

GaAs at  $1\mu\text{m}$ , and assume  $d$  of  $1\mu\text{m}$

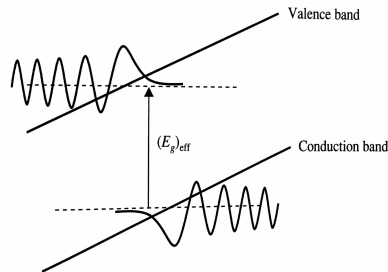
$$V_\pi L \approx 4 [\text{V} \cdot \text{mm}] \Rightarrow \text{Long devices}$$

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## Franz-Keldysh Effect



$$\Delta E_g = \frac{3^{2/3}}{(m^*)^{1/3}} (q\hbar E)^{2/3}$$

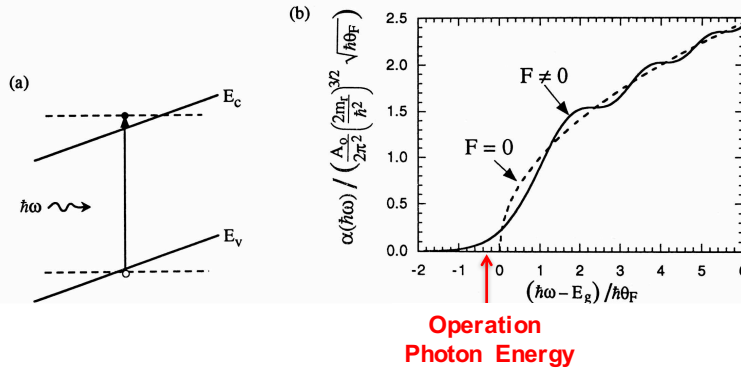
- Under electric field bias, energy bands of electrons are tilted
  - Slope = electric field
- Electron wave functions change from sinusoidal to “Airy” functions
- “Photon-assisted” tunneling
- Effective bandgap becomes smaller
  - Controllable by electric field (voltage)
- Absorption can be modulated by voltage

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## Absorption Modulator with Franz-Keldysh Effect



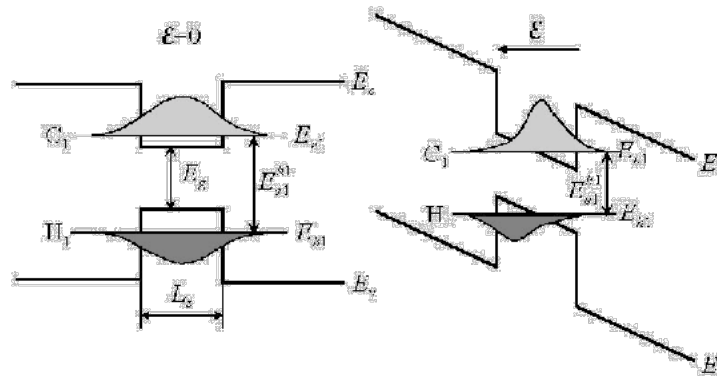
- Absorption edge shifts to lower energy with electric field in direct bandgap semiconductor (e.g., GaAs, InP)
- Absorption up to  $1000 \text{ cm}^{-1}$ 
  - Short devices  $\sim 100 \mu\text{m}$

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## Quantum Confined Stark Effect (QCSE)



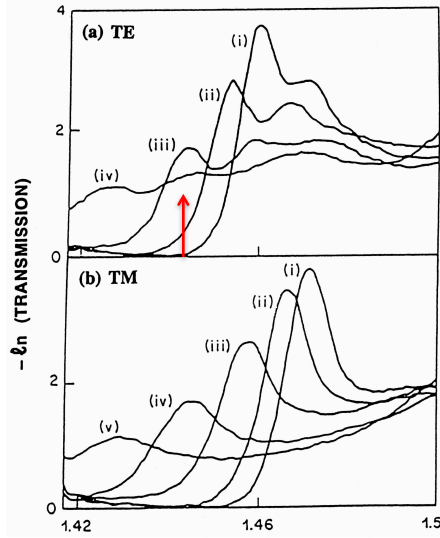
- Absorption edge shifts to lower energy with electric field in quantum wells
- Absorption up to  $1000 \text{ cm}^{-1}$ 
  - Short devices  $\sim 100 \mu\text{m}$

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## Experimentally Measured Absorption in Quantum Wells



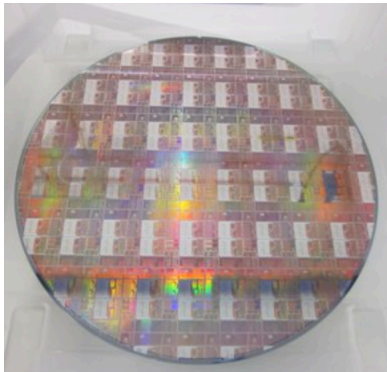
- (i) No field
- (ii) 60 kV/cm
- (iii) 100 kV/cm
- (iv) 150 kV/cm

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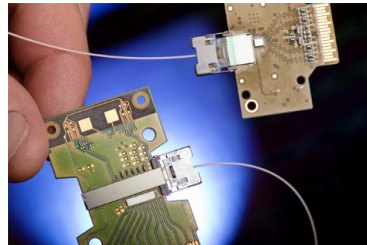
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## Silicon Photonics



<http://www.luxtera.com>



<http://www.intel.com/pressroom/>

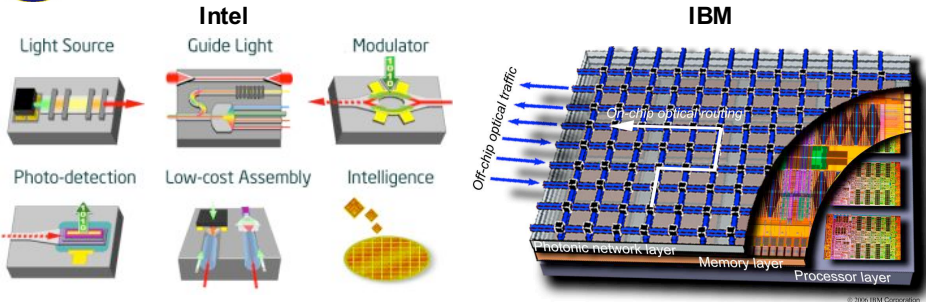
- “Photonics on a silicon chip”
- High density integration
- CMOS process
  - Improved performance
  - Better process control

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# What is Silicon Photonics?



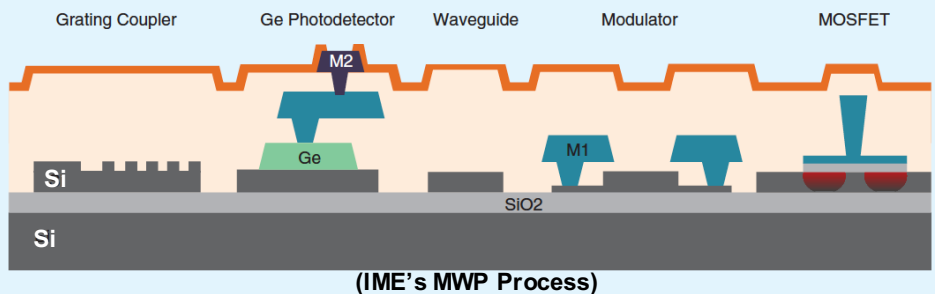
- Use Si to guide, process, and detect (Ge) light
- Leverage on CMOS fab
- Enable photonics to scale with Moore's Law
- Highly functional chip by integrating photonics with CMOS
- Enhance electronic as well as photonic performance

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# Most Common Platform: SOI




M. Hochberg, et al. Solid-State Circuits Mag. IEEE, 2013.

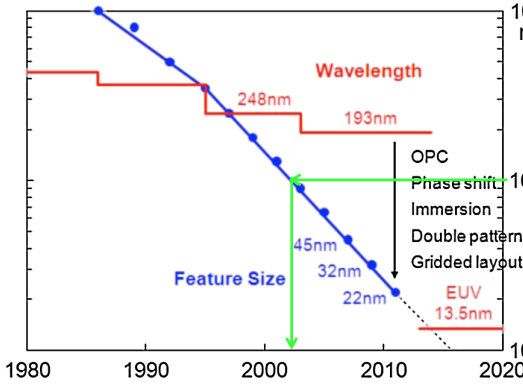
- SOI substrate
  - 220nm Si, 2um BOX
- Si waveguides
- Grating couplers
- Ge detectors
- Si modulators (usually p-i-n)
- Multiple level metals
- (Optional) CMOS

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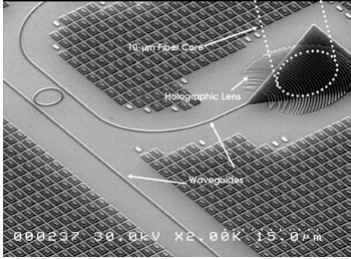
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## Moore's Law on Lithography



### Grating coupler and microring resonator (Luxtera)



**Critical feature ~ 100nm**




**Luxtera**

- Founded in 2001 when lithographic feature size < 1/4 wavelength (~ 100nm)
- First Si Photonics company

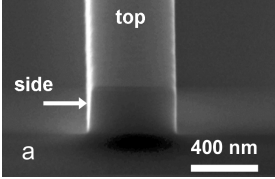
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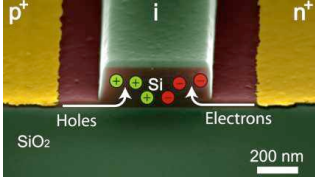
## Properties of Si

### Waveguide



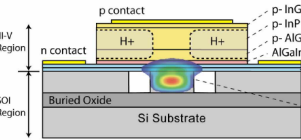
IBM

### Modulator



IBM

### Laser



UCSB / Intel

- + **Low loss** (~ 0.2 to 2 dB/cm)
- + **Tight optical confinement**
  - Tiny waveguides (200x500nm<sup>2</sup>)
  - Sharp bends (radius ~ μm)
- × **Indirect bandgap** → No Si laser
- × **Cubic crystal with inversion symmetry** → No electro-optic effect

- **Optical modulation:**
  - **Modulating carrier density**

$$\Delta n = -8.8 \times 10^{-22} \Delta N_e - 8.5 \times 10^{-18} (\Delta N_h)^{0.8}$$
  - **Thermo-optic effect**

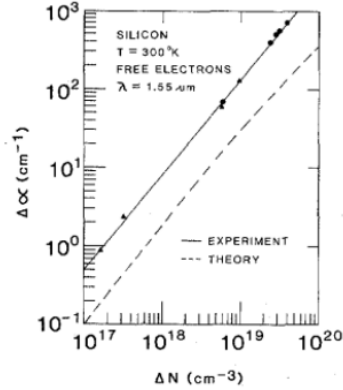
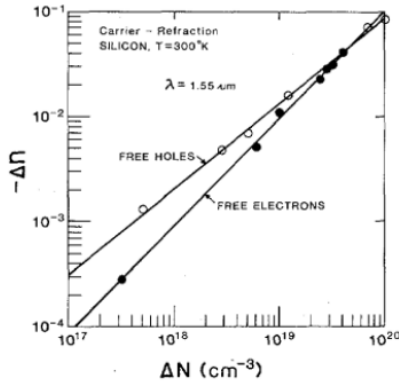
$$\frac{dn}{dT} = 1.86 \times 10^{-4}$$

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## Free-Carrier Effect in Si



At  $1.55\mu\text{m}$ ,

$$\Delta n = -8.8 \times 10^{-22} \Delta N_e - 8.8 \times 10^{-18} (\Delta N_h)^{0.8}$$

$$\Delta \alpha = 8.5 \times 10^{-18} \Delta N_e + 6.0 \times 10^{-18} \Delta N_h$$

For  $\Delta N_e = \Delta N_h = 10^{18} \text{ cm}^{-3}$

$$\Delta n \approx -0.003$$

$$\Delta \alpha \approx 14.5 \text{ cm}^{-1}$$

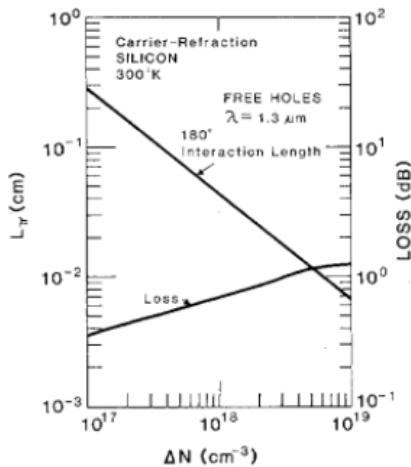
R. A. Soref and B. R. Bennett, "Electrooptical effects in silicon," IEEE Journal of Quantum Electronics, vol. 23, no. 1, pp. 123-129, Jan. 1987.

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## Free-Carrier Effect in Si



- Index change due to free carrier plasma effect
- Index change is small  $\sim 10^{-3}$
- Fundamental trade-off between index change and loss
- Carriers can be changed relatively fast
- $\rightarrow$  Data modulators up to 50Gb/s
- Modulator length is relatively long
  - Need traveling wave electrode/amplifier

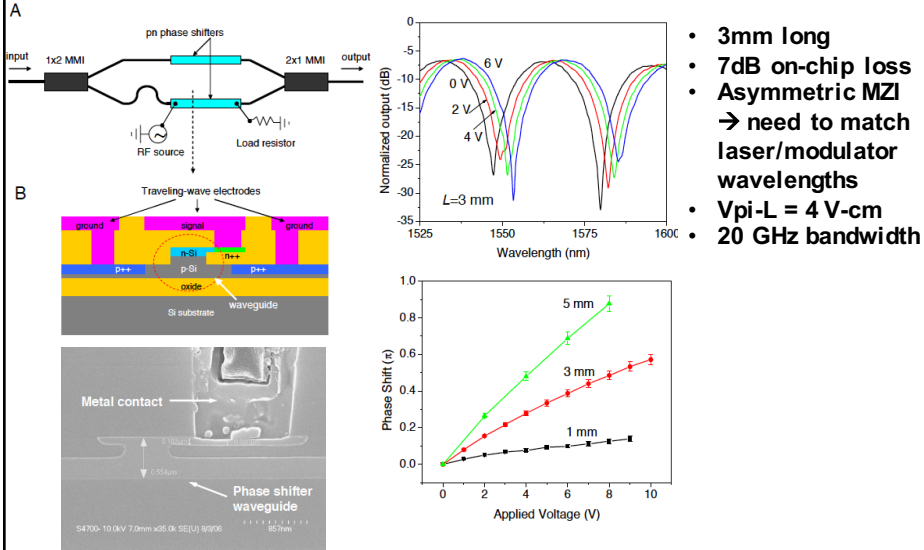
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## Si Mach-Zehnder Modulator



- 3mm long
- 7dB on-chip loss
- Asymmetric MZI  
→ need to match laser/modulator wavelengths
- $V_{\pi} \cdot L = 4 \text{ V-cm}$
- 20 GHz bandwidth

A. Liu, L. Liao, D. Rubin, H. Nguyen, B. Ciftcioglu, Y. Chetrit, N. Izhaky, and M. Paniccia, "High-speed optical modulation based on carrier depletion in a silicon waveguide," *Opt. Express*, vol. 15, no. 2, pp. 660-668, Jan. 2007.

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## Si Mach-Zehnder Modulators

Table 1. Performance comparison of previously reported high-speed silicon MZMs (>25 Gb/s) and devices in this work.

Reference	[17]	[15]	[25]	[27]	[24]	[26] <sup>&amp;</sup>	L = 2 mm	L = 4 mm	L = 6 mm
Device length (mm) <sup>*c</sup>	1	0.12	1.35	1	3.5	2.4	2	4	6
$V_{\pi} \cdot L$ (V-cm)	4	0.5	11	2.8	2.7	2.4	2.4	2.08	1.86
$V_{\pi}$ (V) <sup>*</sup>	NA	NA	NA	NA	~8	10	12	5.2	3.1
Insertion loss (dB) <sup>#</sup>	4	2.5	15	3.7	15	4.3	4.1	6.6	9.0
Speed (Gb/s)	40	25	40	50	40	30	~50	~40	30

<sup>\*c</sup> Device length: the phase shifter length rather than the whole device length.  
<sup>\*</sup>  $V_{\pi}$  under dc. NA represents that the phase shifter is too short and a  $\pi$ -phase shift may not be achievable before breakdown voltage.  
<sup>#</sup> Insertion loss is defined as the on-chip loss for the wavelength at maximum transmission of the MZMs.  
<sup>&</sup> Devices in [26] and in this work are single-drive push-pull MZMs, while the rest are not.

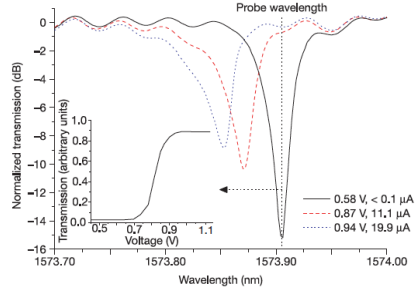
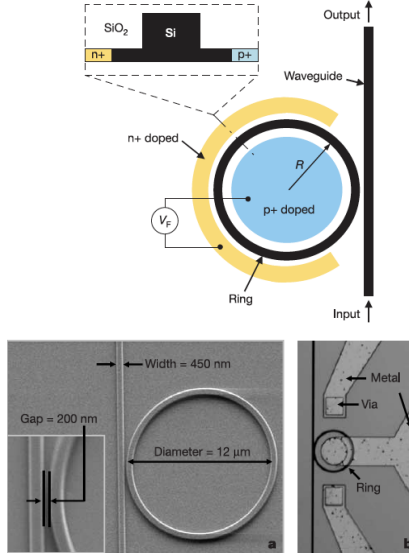
P. Dong, L. Chen, and Y. Chen, "High-speed low-voltage single-drive push-pull silicon Mach-Zehnder modulators," *Optics Express*, vol. 20, no. 6, p. 6163, Mar. 2012.

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## Microring Modulator

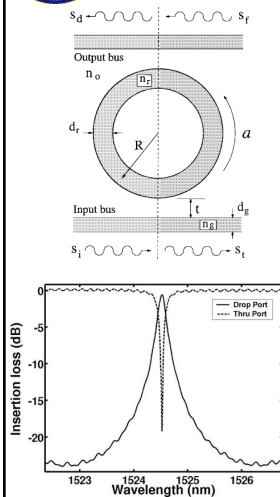


- Use resonance in microring to enhance the modulation
- Reduce modulator size from millimeters to tens of microns
- Enhanced modulation efficiency
- Reduced bandwidth
- Must match laser/resonator wavelengths

Q. Xu, B. Schmidt, S. Pradhan, and M. Lipson, "Micrometre-scale silicon electro-optic modulator," *Nature*, vol. 435, no. 7040, pp. 325–327, May 2005. Prof. Ming Wu



## Basic Physics of Mirroring Resonator



$$S_i = \frac{j(\omega - \omega_0) + \frac{1}{\tau} - \frac{2}{\tau_e}}{j(\omega - \omega_0) + \frac{1}{\tau}} S_i$$

$$|S_r|^2 = \frac{(\omega - \omega_0)^2 + \left(\frac{1}{\tau} - \frac{2}{\tau_e}\right)^2}{(\omega - \omega_0)^2 + \left(\frac{1}{\tau}\right)^2}$$

$$|S_d|^2 = |S_i|^2 - |S_r|^2 = \frac{4}{\tau_e^2} \frac{1}{(\omega - \omega_0)^2 + \left(\frac{1}{\tau}\right)^2}$$

$$\frac{1}{\tau} = \frac{1}{\tau_l} + \frac{1}{\tau_e} + \frac{1}{\tau_d}$$

$\frac{1}{\tau_l}$ : intrinsic loss  
 $\frac{1}{\tau_e}$ : coupling to transmission  
 $\frac{1}{\tau_d}$ : coupling to output (drop)  
 Critical Coupling:  
 $\frac{1}{\tau_e} = \frac{1}{\tau_l} + \frac{1}{\tau_d}$

B. E. Little, S. T. Chu, H. A. Haus, J. Foresi, and J.-P. Laine, "Microring resonator channel dropping filters," *Journal of Lightwave Technology*, vol. 15, no. 6, pp. 998–1005, Jun. 1997.

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## Thermo-Optic Effect in Si

Experimentally measured data at  $1.55\mu\text{m}$ :

$$\frac{dn}{dT} = 9.48 \times 10^{-5} + 3.47 \times 10^{-7} T - 1.49 \times 10^{-10} T^2$$

$$\text{At } 300\text{K}, \frac{dn}{dT} \approx 1.86 \times 10^{-4} \text{ K}^{-1}$$

With  $\Delta T$  of  $270\text{K}$ ,  $\Delta n \approx 0.05$

Corresponding length for  $\pi$  phase shift is  $15.5\mu\text{m}$

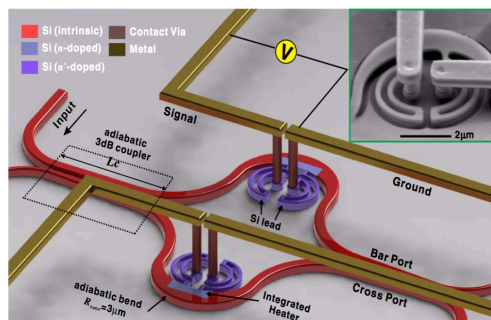
- Index change due to (1) thermal expansion, (2) bandgap energy reduction with temperature
- Relatively strong compared with carrier effect
- Low optical loss introduced by heating
- Usually slow, limited by thermal RC time. Modulation time is on the order of milliseconds. For small structures, it could approach microsecond response time.
- Too slow for modulators, but often used in tunable filters, switches.
- High power consumption

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## Thermo-Optic Effect in Silicon



M. R. Watts, et al., Optics Letters, 2013

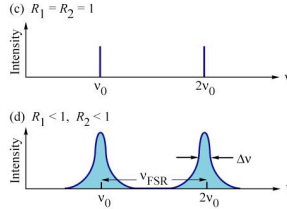
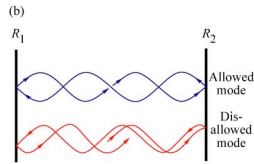
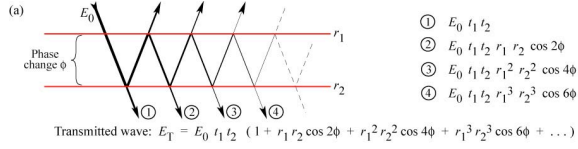
- Si thermo-optic coefficient
  - $dn/dT = 1.86 \times 10^{-4}$
- For  $\Delta T \approx 500^\circ \text{C}$ 
  - $\Delta n \approx 3\%$ ,  $L_\pi \approx 10 \mu\text{m}$
- Steady power consumption
  - $P_\pi \approx 10 \text{ mW}$

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# Fabry-Perot Resonator



$$T = \frac{(1-R)^2}{1+R^2-2R\cos(\Delta\phi)}$$

$$\Delta\phi = 2\beta d = \frac{4\pi n_{eff} d}{\lambda}$$

Fig. 14.1. (a) Transmission of a light wave with electric field amplitude  $E_0$  through a Fabry-Perot resonator. (b) Schematic illustration of allowed and disallowed optical modes in a Fabry-Perot cavity consisting of two coplanar reflectors. Optical mode density for a resonator with (c) no mirror losses ( $R_1 = R_2 = 100\%$ ) and (d) mirror losses.

E. F. Schubert  
 Light-Emitting Diodes (Cambridge Univ. Press)  
 www.LightEmittingDiodes.org